Processing equipment design

Exercise No. 6:
Calculation of conical cover of vessel loaded with internal or external overpressure

EVROPSKÝ SOCIÁLNÍ FOND
PRAHA & EU:INVESTUJEME DO VAŠÍ BUDOUCNOSTI

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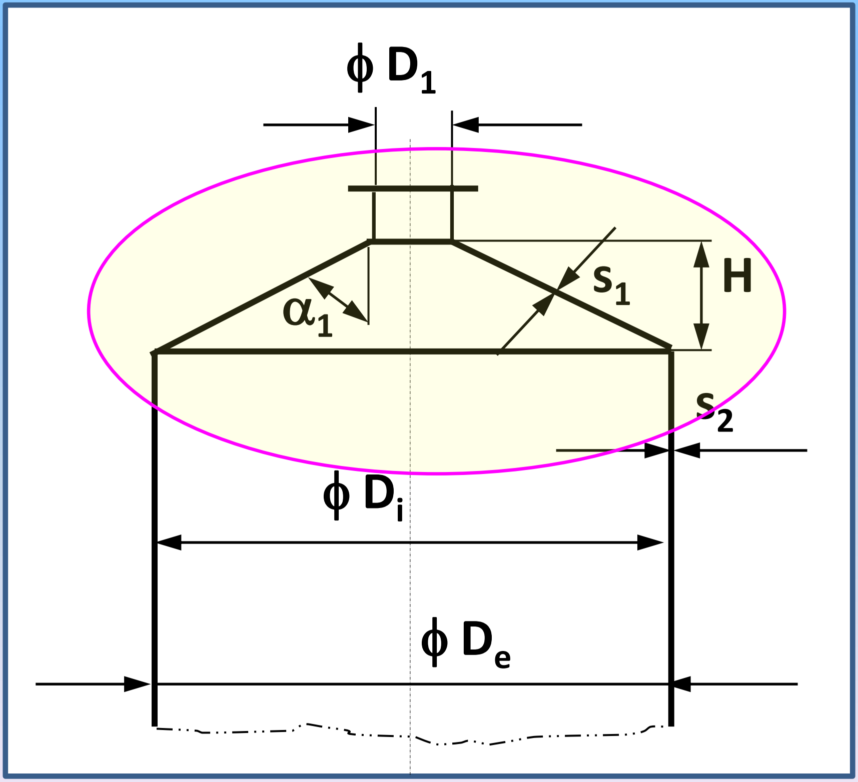
Calculation of conical cover of vessel loaded with internal or external overpressure 100 kPa

(according ČSN 690010, part 4.6)



Sketch of cylindrical vessel with conical cover and neck

1 – cone
2 – cylinder

Given data:

Note:
Following calculations are valid for angle $\alpha_1 \leq 70$ °. We will perform calculations for 2 angels: $\alpha_1 = 45°$ and $\alpha_1 = 70°$.

- **External diameter of cylindrical part** — $D_e = 1500$ mm
- **Realized wall thickness of cylindrical part** — $s_2 = 7$ mm
- **Internal diameter of cylindrical part** — $D_i = D = 1486$ mm
- **Calculating internal/external overpressure** — $p_{i,e} = 100$ kPa
- **Allowed stress of material of cylind. part** — $[\sigma]_1 = \sigma_{D1} = 140$ MPa
- **Allowed stress of material of conical part** — $[\sigma]_2 = \sigma_{D2} = 140$ MPa
- **Modulus of elasticity for both materials** — $E = 206$ GPa
- **Poisson constant for both materials** — $\mu = 0,3$
- **Sum of all allowances (corrosion etc.)** — $c_1 = c_2 = 1$ mm
- **Neck = tube ϕ 159 x 4.5** — $d_1 = 150$ mm
- **Height of conical cover (for $\alpha_1 = 70°$)** — $H \approx 240$ mm
- **Coefficient of longitudinal weld in cone** — $\varphi_W = 0,7$

Internal overpressure 100 kPa – half apex angle α_1 = 45°

Conical shell wall thickness is estimated on the basis of the relationship in ČSN as s_1 = 8.5 mm

$$s_C = s_R / \cos \alpha_1 = (7 - 1) / \cos 45 = 6 / 0.707 \approx 8.5 \text{ mm}$$

Calculated length of transition part (it corresponds to the reach of stress peak)

$$a_1 = 0{,}7 * \sqrt{\frac{D}{\cos\alpha_1} * (s_1 - c)}$$

$$a_1 = 0{,}7 * \sqrt{\frac{1486}{\cos 45} * (8{,}5 - 1)} = 87{,}9 mm$$

$L_t = 0.55*\sqrt{(D*s)}$
$L = 1.65*\sqrt{(D*s)}$

Calculated diameter of conical shell

$$D_C = D - 1{,}4 * a_1 * \sin\alpha_1$$

$$D_C = 1486 - 1.4 * 87.9 * \sin 45 = 1399.0 \text{ mm}$$

Determining whether the calculation formulas given in CSN can be used for our case:

$$0{,}001 \leq \frac{s_1 * cos \propto_1}{D} \leq 0{,}05$$

$$\frac{s_1 * cos \propto_1}{D} = \frac{8{,}5 * cos45}{1486} = 0{,}004 \quad OK$$

The minimum calculated wall thickness of conical shell

$$s_{CR} = \frac{D_C * p}{2 * \sigma_D * \varphi_P - p} * \frac{1}{\cos \alpha_1} = \frac{1399 * 0.100}{2 * 140 * 0.7 + 0.100} * \frac{1}{\cos 45} = 1.01\, mm$$

Maximum internal pressure that can withstand the conical shell

$$[p] = \frac{2 * \sigma_D * \varphi_P * (s_1 - c)}{\frac{D_C}{\cos \alpha_1} + (s_1 - c)} = \frac{2 * 140 * 0.7 * (8.5 - 1)}{\frac{1399}{\cos 45} + (8.5 - 1)} = 0.74\, MPa$$

Internal overpressure 100 kPa – half apex angle $\alpha_1 = 70°$

Conical shell wall thickness is estimated on the basis of the relationship in ČSN again as s_1 = 8.5 mm

Calculated length of transition part (it corresponds to the reach of stress peak)

$$a_1 = 0.7 * \sqrt{\frac{D}{\cos\alpha_1} * (s_1 - c)}$$

$$a_1 = 0.7 * \sqrt{\frac{1486}{\cos 70} * (8.5 - 1)} = 126.3 mm$$

(x for α_1 = 45° it was 87.9 mm)

$\rightarrow$ for > angle is longer reach of stress peak

Calculated diameter of conical shell

$$D_C = D - 1.4 * a_1 * \sin\alpha_1$$

$$D_C = 1486 - 1.4 * 126.3 * \sin 70 = 1320 \text{ mm}$$

(x 1399 mm)

$\rightarrow$ for > angle is < calculated ø of cone

Determining whether the calculation formulas given in CSN can be used for our case:

$$0,001 \le \frac{s_1 * \cos \alpha_1}{D} \le 0,05$$

$$\frac{s_1 * \cos \alpha_1}{D} = \frac{8,5 * \cos 70}{1486} = 0,002 \quad OK$$

The minimum calculated wall thickness of conical shell

$$s_{CR} = \frac{D_C * p}{2 * \sigma_D * \varphi_P - p} * \frac{1}{\cos \alpha_1} = \frac{1320 * 0,100}{2 * 140 * 0.7 + 0.100} * \frac{1}{\cos 70} = 1.97\ mm$$

(x 1.01 mm)

Maximum internal pressure that can withstand the conical shell

$$[p] = \frac{2 * \sigma_D * \varphi_P * (s_1 - c)}{\frac{D_C}{\cos \alpha_1} + (s_1 - c)} = \frac{2 * 140 * 0.7 * (8.5 - 1)}{\frac{1399}{\cos 70} + (8.5 - 1)} = 0.36\ MPa$$

(x 0.74 MPa)

External overpressure 100 kPa – half apex angle α_1 = 45°

Preliminary determination of wall thickness s_1

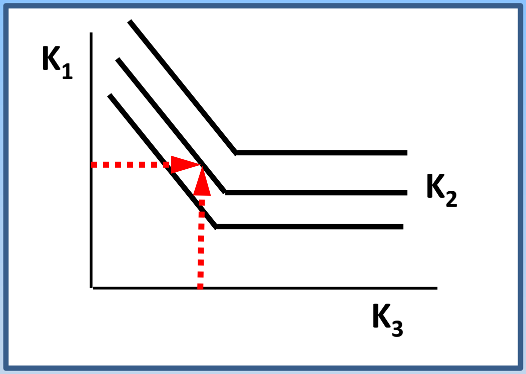
= the real situation in which the heater has worked

The wall thickness is specified according equations valid for cylinder (part 4.5) multiplied by value $1/\cos\alpha_1$.

In our previous example of the cylindrical vessel loaded with external pressure we specified the cylinder thickness without allowances:

$$s_{R2} = Max\left\{K_2 * D * 10^{-2}; \frac{1.1 * p * D}{2 * \sigma_D}\right\} = 4.80mm$$

$$K_1 = \frac{n_U * p}{2{,}4 * 10^{-6} * E} \qquad K_3 = \frac{L}{D} \longrightarrow K_2 = \frac{100 * (s - c)}{D}$$



Then is the preliminary wall thickness of the conical part

$$s_{R1} = \frac{s_{R2}}{\cos\alpha_1} = \frac{4.8}{\cos 45} = 6.8\ mm$$

Preliminary designed wall thickness together with all allowances is

$s_1 = s_{R1} + c = 6.8 + 1 \approx 8.0$ mm

Calculating diameter of a smooth conical shell without torus in connection with the cylinder (used in next calculations)

$D_C = D - 1.4 * a_1 * \sin\alpha_1$

where is

$$a_1 = 0.7 * \sqrt{\frac{D}{\cos\alpha_1} * (s_1 - c)}$$

Calculated length of transition part (it corresponds to the reach of stress peak)

$$a_1 = 0.7 * \sqrt{\frac{1486}{\cos 45} * (8.0 - 1)} = 84.9mm$$

(x for internal pressure and 45° it was 87.9 mm)

$D_C = 1486 - 1.4 * 84.9 * \sin 45 = 1402$ mm *(x 1399 mm)*

Checking of the preliminary specified wall thickness for the external overpressure

Allowed external overpressure

$$[p] = \frac{[p]_P}{\sqrt{1 + \left(\frac{[p]_P}{[p]_E}\right)^2}}$$

Allowed external overpressure in plastic state (region)

(similar relationship to internal pressure = achieving of allowable stress in the shell)

$$[p]_P = \frac{2 * \sigma_D * (s_1 - c)}{\frac{D_C}{\cos\alpha_1} + (s_1 - c)} = \frac{2 * 140 * (8.0 - 1)}{\frac{1402}{\cos 45} + (8.0 - 1)} = 0.985 MPa$$

$$[p_P] = \frac{2 * [\sigma_D] * (s - c)}{D + (s - c)}$$

D $\rightarrow$ D_K / cos α_1

Allowed external overpressure in elastic state (region)

(stability loss in the elastic region, it is for $\sigma < \sigma_D$)

$$[p]_E = \frac{20.8 * 10^{-6} * E}{n_U * B_1} * \frac{D_E}{l_E} * \left[\frac{100 * (s_1 - c)}{D_E}\right]^2 * \sqrt{\frac{100 * (s_1 - c)}{D_E}}$$

For cylinder:

D $\rightarrow$ D_E
l $\rightarrow$ l_E

n_U = 2,4 ... safety coefficient against a stability loss (see cylinder) where effective dimensions of a conical shell are:

$$l_E = \frac{D - d_1}{2 * \sin \alpha_1} = \frac{1486 - 150}{2 * \sin 45} = 944.7 mm$$

effective length of conical shell

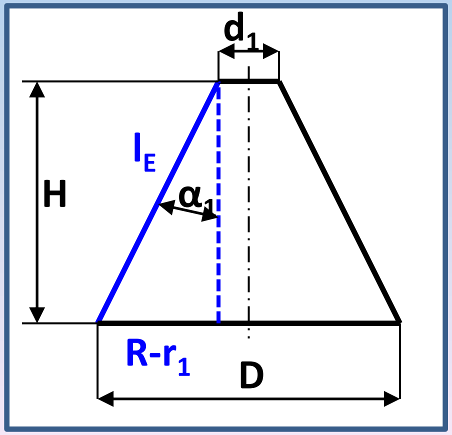
$$D_E = Max\{D_{E1}; D_{E2}\}$$

effective diameter of conical shell

$$D_{E1} = \frac{D + d_1}{n_U * \cos \alpha_1} = \frac{1486 + 150}{2.4 * \cos 45} = 1156.8 mm$$

D – higher diameter
d_1 - lower diameter

$$D_{E2} = \frac{D}{\cos \alpha_1} - 0.31 * (D + d_1) * \sqrt{\frac{D + d_1}{s_1 - c}} * tg \alpha_1$$



$$D_{E2} = \frac{1486}{\cos 45} - 0.31 * (1486 + 150) * \sqrt{\frac{1486 + 150}{8.0 - 1}} * tg 45 = -5651. mm$$

(for $\alpha_1 = 0 \rightarrow D_{E1} = (D+d_1)/n_u < D; D_{E2} = D$)

D_E = 1156.8 mm

Parameter B1 is specified from equation

$$B_1 = Min\left\{B_{11} = 1.0; B_{12} = 9{,}45 * \frac{D_E}{l_E} * \sqrt{\frac{D_E}{100*(s_1 - c)}}\right\}$$

like for the cylinder

$$B_{12} = 9.45 * \frac{1156.8}{944.7} * \sqrt{\frac{1156.8}{100*(8.0-1)}} = 15.0$$

B_1 = 1.0

Allowed external overpressure in elastic state (region) is

$$[p]_E = \frac{20.8*10^{-6}*206*10^3}{2.4*1{,}0} * \frac{1156.8}{944.7} * \left[\frac{100*(8.0-1)}{1156.8}\right]^2 * \sqrt{\frac{100*(8.0-1)}{1156.8}}$$

$$[p]_E = 0.622\ MPa$$

Then is the maximal allowed external overpressure for the preliminary specified cone wall thickness s_1 = 8.0 mm

$$[p] = \frac{0.985}{\sqrt{1+\left(\frac{0.985}{0.622}\right)^2}} = 0.526\ MPa \geq 0.100\ MPa$$

Such conical shell is too overdesigned. Therefore we will repeat these calculations for a newly estimated cone wall thickness.

Note:

When calculating the cylindrical part of the vessel to the external overpressure the calculated wall thickness was s_R = 4.8 mm, it means that the realized wall thickness (with allowances for corrosion etc.) was about 6 mm.

b) New estimation of the preliminary designed wall cone thickness together with all allowances

(x 8 mm)

$s_1 = s_{R1} + c = 4 + 1 \approx 5$ mm

(results of the last iteration of the cone wall thickness calculation)

Calculating diameter of a smooth conical shell without torus in connection with the cylinder (used in next calculations)

$D_C = D - 1.4 * a_1 * \sin\alpha_1$

where is

$$a_1 = 0.7 * \sqrt{\frac{D}{\cos\alpha_1} * (s_1 - c)}$$

Calculated length of transition part (it corresponds to the reach of stress peak)

$$a_1 = 0.7 * \sqrt{\frac{1486}{\cos 45} * (5 - 1)} = 64.1 mm$$

(thinner wall → shorter length of transition part)

(x 84.9 mm)

D_C = 1486 – 1.4 * 64.1 * sin45 = 1422.5 mm *(x 1156.8 mm)*

Checking of the preliminary specified wall thickness for the external overpressure

Allowed external overpressure

$$[p]=\frac{[p]_P}{\sqrt{1+\left(\frac{[p]_P}{[p]_E}\right)^2}}$$

Allowed external overpressure in plastic state (region)

$$[p]_P=\frac{2*\sigma_D*(s_1-c)}{\frac{D_C}{\cos\alpha_1}+(s_1-c)}=\frac{2*140*(5-1)}{\frac{1422.5}{\cos 45}+(5-1)}=0.556MPa$$

(x 0.985 MPa)

Allowed external overpressure in elastic state (region)

$$[p]_E=\frac{20.8*10^{-6}*E}{n_U*B_1}*\frac{D_E}{l_E}*\left[\frac{100*(s_1-c)}{D_E}\right]^2*\sqrt{\frac{100*(s_1-c)}{D_E}}$$

where effective dimensions of a conical shell are

$$l_E=\frac{D-d_1}{2*\sin\alpha_1}=\frac{1486-150}{2*\sin 45}=944.7mm$$

$$D_E=Max\{D_{E1};D_{E2}\}$$

$$D_{E1} = \frac{D + d_1}{n_U * \cos\alpha_1} = \frac{1486 + 150}{2.4 * \cos 45} = 1156.8 mm$$

$$D_{E2} = \frac{D}{\cos\alpha_1} - 0.31 * (D + d_1) * \sqrt{\frac{D + d_1}{s_1 - c}} * tg\alpha_1$$

$$D_{E2} = \frac{1486}{\cos 45} - 0.31 * (1486 + 150) * \sqrt{\frac{1486 + 150}{5 - 1}} * tg 45 = -9205.9 mm$$

D_E = 1156.8 mm ***(x 1402 mm)***

$$B_1 = Min\left\{ B_{11} = 1.0; B_{12} = 9.45 * \frac{D_E}{l_E} * \sqrt{\frac{D_E}{100 * (s_1 - c)}} \right\}$$

$$B_{12} = 9.45 * \frac{1156.8}{944.7} * \sqrt{\frac{1156.8}{100 * (5 - 1)}} = 19.7$$

B_1 = 1.0

$$[p]_E = \frac{20.8*10^{-6}*206*10^3}{2.4*1.0}*\frac{1156.8}{944.7}*\left[\frac{100*(5-1)}{1156.8}\right]^2*\sqrt{\frac{100*(5-1)}{1156.8}}$$

$$[p]_E = 0.154\,MPa$$

(x 0.622 MPa)

Then is the maximal allowed external overpressure for this specified cone wall thickness s_1 = 5 mm

$$[p] = \frac{0.556}{\sqrt{1+\left(\frac{0.556}{0.154}\right)^2}} = 0.148 MPa \geq 0.100 MPa$$

(x for s_1 = 8 mm it was 0.526 MPa)

Note:

For s_1 = 4.5 mm is the maximal external pressure [p] = 0.107 MPa = 107 kPa. The value is practically equal to the given external overpressure.

For the cylinder was the wall thicknes s_2 = 4.8 mm.

External overpressure 100 kPa – half apex angle α_1 = 70 °.

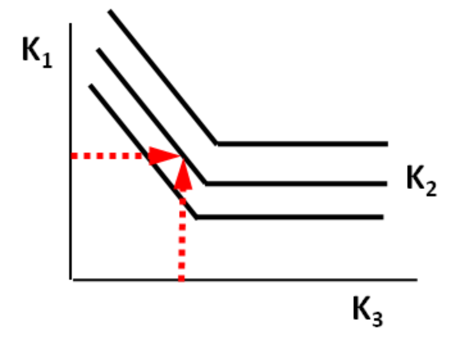
Preliminary specification of wall thickness s_1

The wall thickness is specified according equations valid for cylinder (part 4.5) multiplied by value $1/\cos\alpha_1$.

In our previous example of the cylindrical vessel loaded with external pressure we specified the cylinder thickness without allowances:

$$s_{R2} = Max\left\{K_2 * D * 10^{-2}; \frac{1.1 * p * D}{2 * \sigma_D}\right\} = 4.80mm$$

$$K_1 = \frac{n_U * p}{2{,}4 * 10^{-6} * E} \qquad K_3 = \frac{L}{D} \longrightarrow K_2 = \frac{100 * (s - c)}{D}$$



Then is the preliminary wall thickness of the conical part

$$s_{R1} = \frac{s_{R2}}{\cos\alpha_1} = \frac{4.8}{\cos 70} = 14.04\ mm$$

For α_1 = 45 ° it was s_{R1} = 6.8 mm
→ disadvantage of flat cones

Preliminary designed wall thickness together with all allowances is

$s_1 = s_{R1} + c = 14.04 + 1 \approx 15.0$ mm

Calculating diameter of a smooth conical shell without torus in connection with the cylinder (used in next calculations)

$D_K = D - 1.4 * a_1 * \sin\alpha_1$

where is

$$a_1 = 0.7 * \sqrt{\frac{D}{\cos\alpha_1} * (s_1 - c)}$$

Calculated length of transition part
(it corresponds to the reach of stress peak)

$$a_1 = 0.7 * \sqrt{\frac{1486}{\cos 70} * (15.0 - 1)} = 172.6 mm$$

(x for 45° and s_1 = 8 mm was a_1 = 84.6 mm)

$D_C = 1486 - 1.4 * 172.6 * \sin 70 = 1259$ mm (x 1402 mm)

Checking of the preliminary specified wall thickness for the external overpressure

Allowed external overpressure

$$[p]=\frac{[p]_P}{\sqrt{1+\left(\frac{[p]_P}{[p]_E}\right)^2}}$$

Allowed external overpressure in plastic state (region)

(similar relationship to internal pressure = achieving of allowable stress in the shell)

$$[p]_P=\frac{2*\sigma_D*(s_1-c)}{\frac{D_C}{\cos\alpha_1}+(s_1-c)}=\frac{2*140*(15.0-1)}{\frac{1259}{\cos 70}+(15.0-1)}=1.061\,MPa$$

(x for 45° and estimation 8 mm it was 0.556 MPa)

Allowed external overpressure in elastic state (region)

(stability loss in the elastic region, it is for $\sigma < \sigma_D$)

$$[p]_E=\frac{20.8*10^{-6}*E}{n_U*B_1}*\frac{D_E}{l_E}*\left[\frac{100*(s_1-c)}{D_E}\right]^2*\sqrt{\frac{100*(s_1-c)}{D_E}}$$

n_U = 2,4 ... **safety coefficient against a stability loss (see cylinder) where effective dimensions of a conical shell are:**

$$l_E = \frac{D-d_1}{2*\sin\alpha_1} = \frac{1486-150}{2*\sin 70} = 710.9mm$$

effective length of conical shell x 944.7 mm

$$D_E = Max\{D_{E1}; D_{E2}\}$$

effective diameter of conical shell x 1156.8 mm

$$D_{E1} = \frac{D+d_1}{n_U * \cos\alpha_1} = \frac{1486+150}{2.4*\cos 70} = 2392mm$$

D – higher diameter
d_1 - lower diameter

$$D_{E2} = \frac{D}{\cos\alpha_1} - 0.31*(D+d_1)*\sqrt{\frac{D+d_1}{s_1-c}}*tg\alpha_1$$

$$D_{E2} = \frac{1486}{\cos 70} - 0.31*(1486+150)*\sqrt{\frac{1486+150}{15.0-1}}*tg70 = -12890mm$$

D_E = 2392 mm

Parameter B_1 is specified from equation

$$B_1 = Min\left\{B_{11} = 1.0; B_{12} = 9.45 * \frac{D_E}{l_E} * \sqrt{\frac{D_E}{100*(s_1 - c)}}\right\}$$

like for the cylinder

$$B_{12} = 9.45 * \frac{2392}{710.9} * \sqrt{\frac{2392}{100*(15.0-1)}} = 41.56$$

B_1 = 1.0

Allowed external overpressure in elastic state (region) is

$$[p]_E = \frac{20.8*10^{-6}*206*10^3}{2.4*1.0} * \frac{2392}{710.9} * \left[\frac{100*(15-1)}{2392}\right]^2 * \sqrt{\frac{100*(15-1)}{2392}}$$

$$[p]_E = 1.574\ MPa$$

(x for 45° it was 0.622 MPa)

Then is the maximal allowed external overpressure for the preliminary specified cone wall thickness s_1 = 15 mm

$$\left[p\right]=\frac{1.061}{\sqrt{1+\left(\frac{1.061}{1.574}\right)^2}}=0.880\ MPa\geq 0.100\ MPa$$

$\rightarrow$ **8.8 x overdesigned preliminary estimation**

x 0.526 MPa (for 45°)

$\rightarrow$ **5.3 x overdesigned preliminary estimation**

Such conical shell is for the wall thickness s_1 = 15 mm again too overdesigned. Therefore we will repeat these calculations for a newly estimated cone wall thickness.

Note:
When calculating the cylindrical part of the vessel to the external overpressure the calculated wall thickness was s_R = 4.8 mm, it means that the realized wall thickness (with allowances for corrosion etc.) was about 6 mm.

b) New estimation of the preliminary designed wall cone thickness together with all allowances

$s_1 = s_{R1} + c = 5 + 1 \approx 6$ mm (x for 45° it was $s_1 = 5.0$ mm)

Calculating diameter of a smooth conical shell without torus in connection with the cylinder (used in next calculations)

$D_C = D - 1.4 * a_1 * \sin\alpha_1$

where is

$$a_1 = 0.7 * \sqrt{\frac{D}{\cos\alpha_1} * (s_1 - c)}$$

$$a_1 = 0.7 * \sqrt{\frac{1486}{\cos 70} * (6 - 1)} = 103.2 mm$$

(x for 45° it was 64.1 mm)

D_C = 1486 – 1.4 * 103.2 * sin70 = 1350 mm (x 1422.5 mm)

Checking of the preliminary specified wall thickness for the external overpressure

Allowed external overpressure

$$[p] = \frac{[p]_P}{\sqrt{1 + \left(\frac{[p]_P}{[p]_E}\right)^2}}$$

Allowed external overpressure in plastic state (region)

$$[p]_P = \frac{2 * \sigma_D * (s_1 - c)}{\frac{D_K}{\cos\alpha_1} + (s_1 - c)} = \frac{2*140*(6-1)}{\frac{1350}{\cos 70} + (6-1)} = 0.354 MPa$$

x for 70° and preliminary estimation s_1 = 15 mm it was 0.556 MPa

Allowed external overpressure in elastic state (region)

$$[p]_E = \frac{20{,}8 * 10^{-6} * E}{n_U * B_1} * \frac{D_E}{l_E} * \left[\frac{100 * (s_1 - c)}{D_E}\right]^2 * \sqrt{\frac{100 * (s_1 - c)}{D_E}}$$

where effective dimensions of a conical shell are:

$$l_E = \frac{D - d_1}{2 * \sin\alpha_1} = \frac{1486 - 150}{2 * \sin 70} = 710.9 mm$$

$$D_E = Max\{D_{E1}; D_{E2}\}$$

$$D_{E1} = \frac{D + d_1}{n_U * \cos\alpha_1} = \frac{1486 + 150}{2.4 * \cos 70} = 2392mm$$

$$D_{E2} = \frac{D}{\cos\alpha_1} - 0.31 * (D + d_1) * \sqrt{\frac{D + d_1}{s_1 - c}} * tg\,\alpha_1$$

$$D_{E2} = \frac{1486}{\cos 70} - 0{,}31 * (1486 + 150) * \sqrt{\frac{1486 + 150}{6 - 1}} * tg\,70 = -23033mm$$

D_E = 2392 mm

(x 1156.8 mm)

$$B_1 = Min\left\{ B_{11} = 1.0; B_{12} = 9.45 * \frac{D_E}{l_E} * \sqrt{\frac{D_E}{100 * (s_1 - c)}} \right\}$$

$$B_{12} = 9.45 * \frac{2392}{710.9} * \sqrt{\frac{2392}{100 * (6 - 1)}} = 69.5$$

B_1 = 1.0

$$[p]_E = \frac{20.8*10^{-6}*206*10^3}{2.4*1.0}*\frac{2392}{710.9}*\left[\frac{100*(6-1)}{2392}\right]^2*\sqrt{\frac{100*(6-1)}{2392}}$$

$$[p]_E = 0.120\, MPa$$

(x for 45° and s_1 = 5.0 mm it was 0.154 MPa)

Then is the maximal allowed external overpressure for this specified cone wall thickness s_1 = 6 mm

$$[p] = \frac{0.354}{\sqrt{1+\left(\frac{0.354}{0.120}\right)^2}} = 0.114 MPa \geq 0.100 MPa$$

(x for 45° and s_1 = 5.0 mm it was 0.148 MPa)

(x for 70° and s_1 = 15.0 mm it was 0.880 MPa)

Realized wall thickness s_1 = 8 mm of the conical cover is sufficient for this higher angle too.

Summary of results

half apex angle	internal overpressure 100 kPa	external overpressure 100 kPa
	calculated wall thickness (mm)	
$\alpha = 0°$ (cylinder)	0.54	4.8
$\alpha = 45°$ (cone)	1.0	3.5
$\alpha = 70°$ (cone)	2.0	5.0

Note:

- *From the viewpoint of internal overpressure is preferable cylindrical vessel.*
- *From the viewpoint of external overpressure (stability = collapse) is preferable sharper cone*

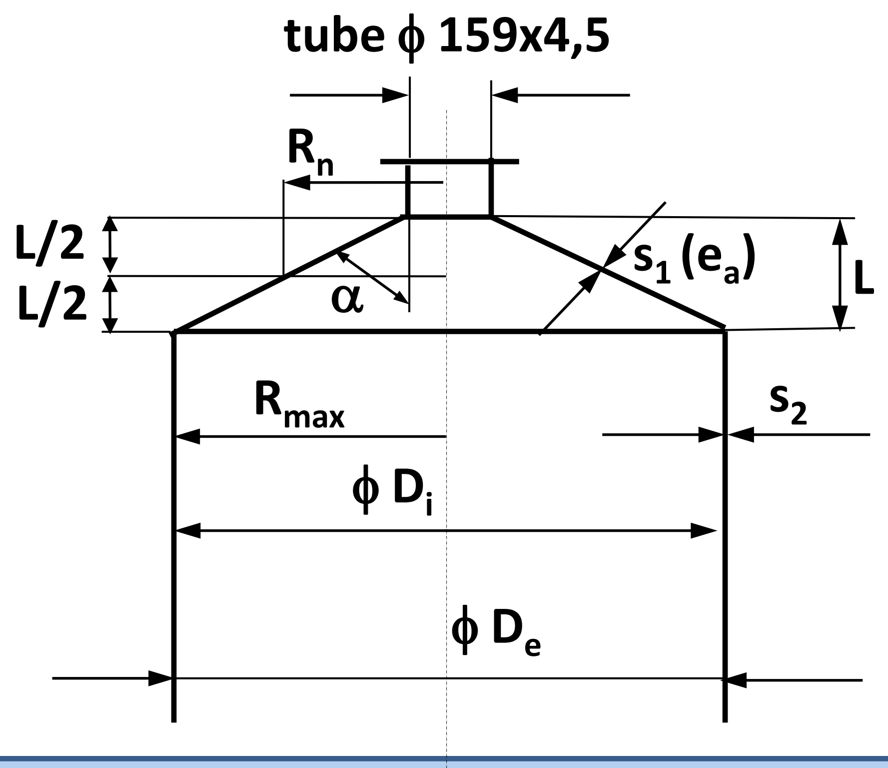
Design of conical bottom for $\alpha = 70\,°$ according European standard ES 13445-3

These calculations are performed according the ES 13445-3, part 8.6.3. that was several times amended. The last change was in 2010.

These calculations are valid for $\alpha \le 75\,°$.

Because the standard uses quite different symbols I show the sketch of the conical cover again with these new symbols.

Characteristic dimensions



Internal radius of cylindrical shell

$$R_{max} = D_i / 2 = 1486 / 2 = 743 \text{ mm}$$

(as in the case of ES for cylinder is not the analyzed thickness calculated, but is estimated)

Analyzed wall thickness of the conical shell without allowances

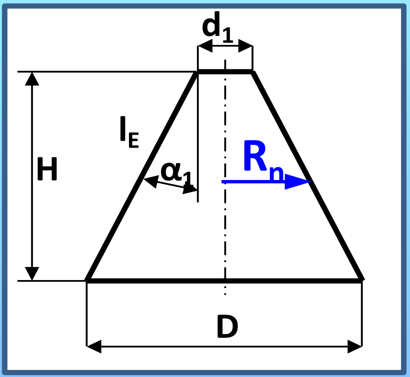
$$e_a = s_1 - c = 6 - 1 = 5 \text{ mm}$$

(it corresponds to the calculated thickness)

(we consider the wall thicknes specified according the ČSN, that is checked according the EN)

Mean radius of the conical shell

$R_n = (R_{max} + R_{TR}) / 2 = (743 + 150 / 2) / 2 = 409$ mm



Height of the conical shell

$L = 240$ mm

$L = H$;
$R_{max} = D_i/2$;
$R_{TR} = d_{1i}/2$

Check of the shell against a stability loss between two reinforcements (it is cylindrical parts = shell and neck).

(the cone is without any reinforcement → L = H)

According part. 8.4.3 (eq. 8.4.3-1) is the allowed elastic limit

$\sigma_e = R_{P0.2/t} / 1.25 = 210 / 1.25 = 168$ MPa (safety coefficient 1.25)

where $R_{P0.2/t} = R_{emin} = 210$ MPa is the yield point for working temperature.

(x according ČSN is $\sigma_D = 140$ MPa)

Then is a pressure P_Y, at what a mean tangential stress reaches the yield point in the center of shell between reinforcements

$$P_Y = (e_a * \sigma_e * \cos\alpha) / R_{max}$$

$$P_Y = (5 * 168 * \cos 70) / 743 = 0.387 \text{ MPa}$$

$s = p*R/\sigma_D*\cos\alpha_1 \rightarrow$
$p = s*\sigma_D*\cos\alpha_1/R$

x 0.354 MPa

Now we can specify a theoretic pressure for an elastic loss of stability of the shell wall

$$P_m = (E * e_a * \varepsilon * \cos^3\alpha) / R_n$$

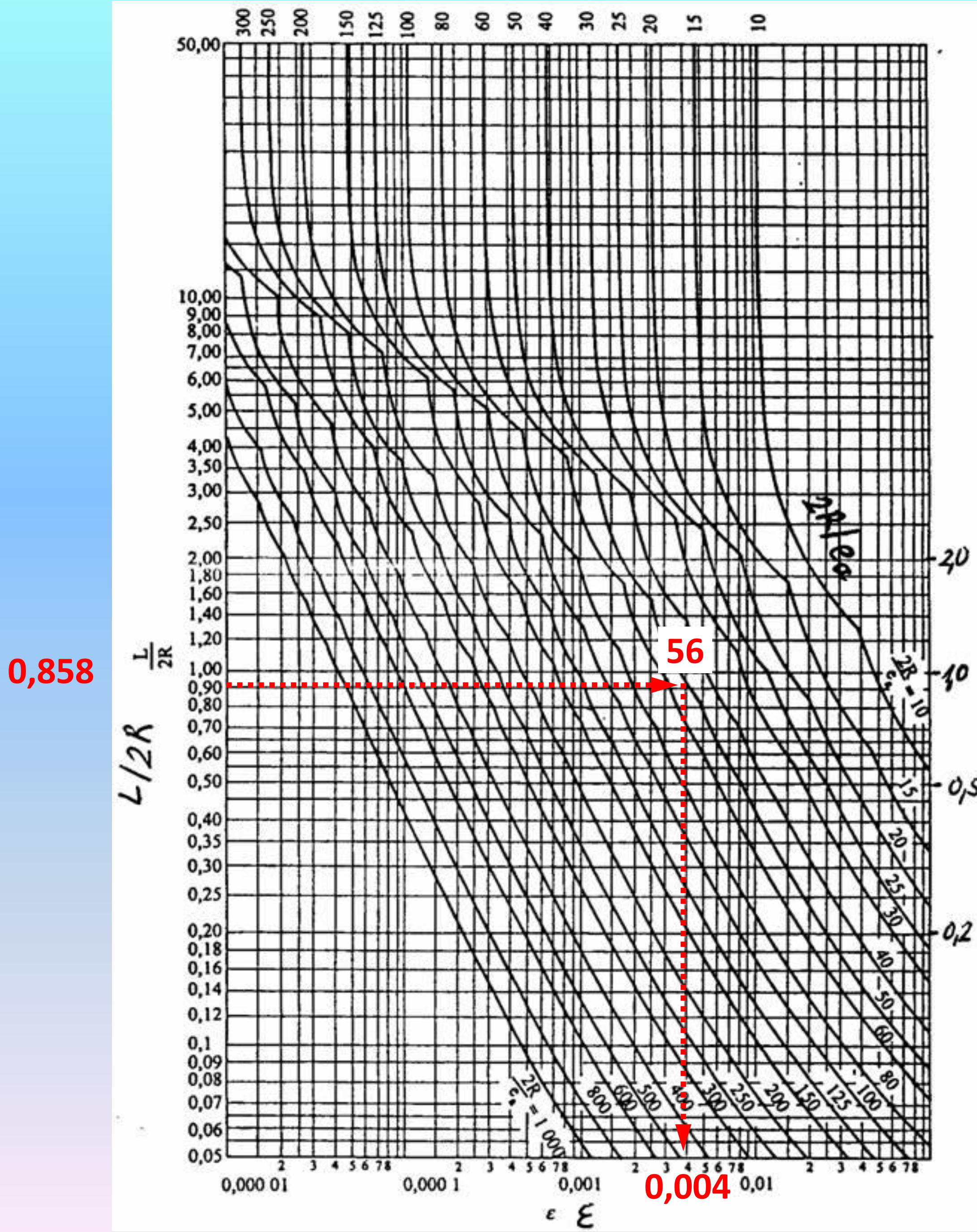
where the value ε is specified from fig. 8.5-3 (see the next page) for

$$L / (2 * R_n * \cos\alpha) = 240 / (2 * 409 * \cos 70) = 0.858$$

instead (L/2R) for cylinder in the ČSN (dimensionless length) **and**

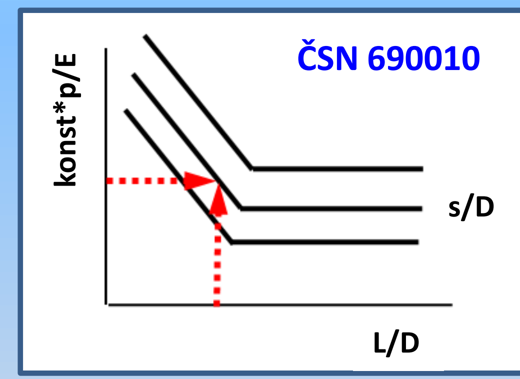
$$(2 * R_n * \cos\alpha) / e_a = (2 * 409 * \cos 70) / 5 = 55.95$$

instead (s/R) for cylinder in the ČSN (dimensionless wall thickness) .



$2R/e_a$ = reciprocal value of dimensionless thickness of the wall

x according ČSN



It follows from the diagram

$\varepsilon \approx 0{,}004$

Then we can specify the value P_m
= theoretical pressure at the elastic stability loss of conical shell

$$P_m = (206000 * 5 * 0{,}004 * \cos^3 70) / 409 = 0.403 \text{ MPa}$$

E (MPa) s (mm) ε (-) R_n (mm)

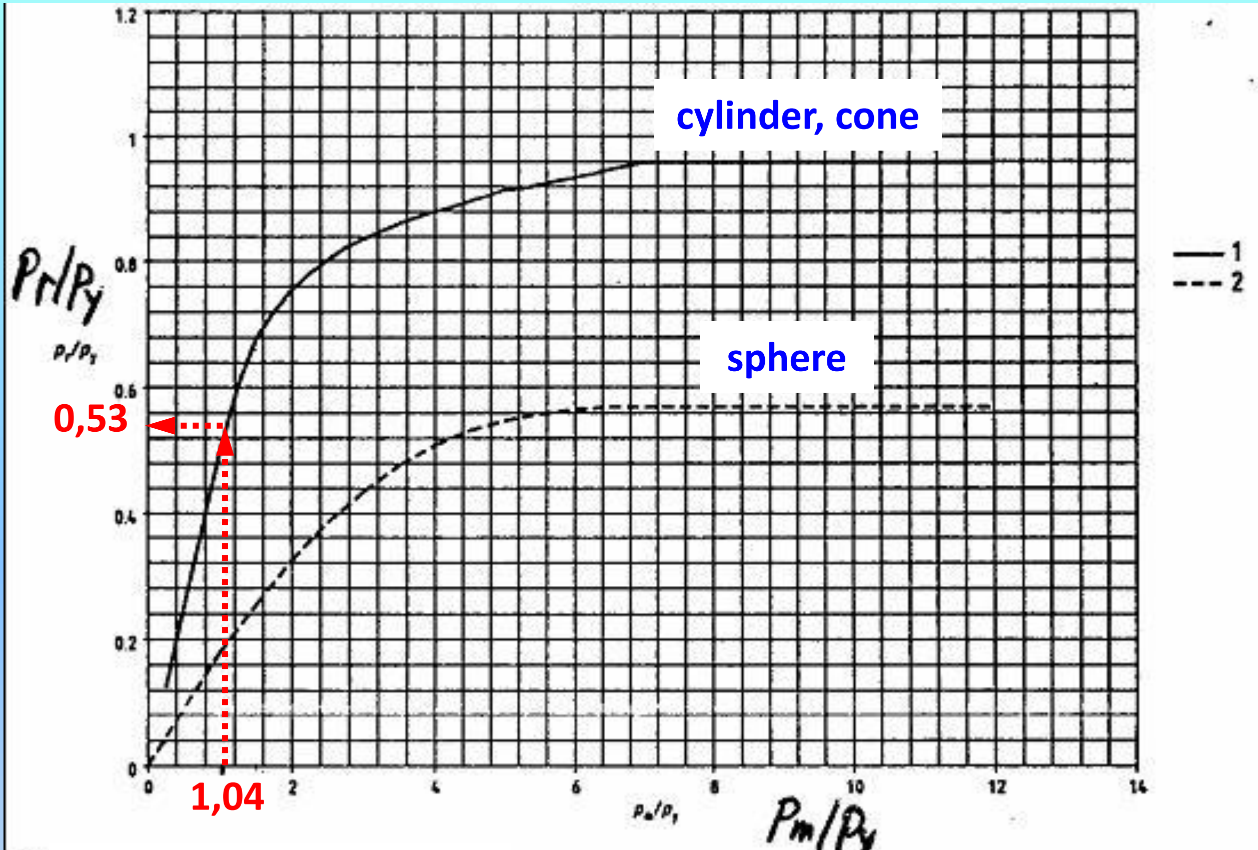
Now we specify a ratio $P_m / P_Y = 0.403 / 0.387 = 1.04$.

(teor. pressure at elastic stability loss / teor. pressure when is reached allowed stress)

For curve 1 from following diagram 8.5-5 a ratio $P_r / P_y \approx 0{,}53$ is found and from it the low limit of the calculating external overpressure for the stability loss.

$$P_r = 0.53 * P_Y = 0.53 * 0.387 = 0.205 \text{ MPa}$$

(similar way like was for the cylinder)



$= -0{,}0569x^3 + 0{,}077x^2 +$
$+ 0{,}4769x + 0{,}0005$

for Pm/Py = 0-1,25

1 - cylindrical and conical shells

P_m/P_y	0	0,25	0,5	0,75	1,0	1,25	1,5	1,75	2,0	2,25	2,5	2,75	3	3,25	3,5
P_r/P_y	0	0,125	0,251	0,375	0,5	0,605	0,68	0,72	0,755	0,78	0,803	0,822	0,836	0,849	0,861

P_m/P_y	3,75	4,0	4,25	4,5	4,75	5,0	5,25	5,5	5,75	6,0	6,25	6,5	6,75	≥ 7,0
P_r/P_y	0,87	0,879	0,887	0,896	0,905	0,914	0,917	0,923	0,929	0,935	0,941	0,947	0,953	0,959

2 - spherical shells and vaulted covers

P_m/P_y	0	0,5	1	1,5	2	2,5	3,0	3,5	4	4,5	5,0	5,5	6	≥ 6,5
P_r/P_y	0	0,09	0,18	0,255	0,324	0,386	0,435	0,479	0,51	0,533	0,548	0,565	0,567	0,57

Fig. 8.5-5 – Values Pr/Py as function of Pm/Py

Maximal calculating overpressure for the realized wall thickness must conform to the condition (8.6.3-5)

$$P \leq P_r \,/\, S$$

where S = 1.5 is safety coefficient according part 8.4.4

$$P \leq 0{,}205 \,/\, 1.5 = 0.137 \text{ MPa}$$

Real maximal theoretic external overpressure is 0.100 MPa. From it follows that according the ES is the conical shell O.K.

Comparison of results for $\alpha_1 = 70°$

	calculated wall thickness	max. external overpressure
According new ČSN	s = 6 mm	0,114 MPa
According old ČSN	s = 5.4 mm	0,100 MPa
	s = 6 mm	0,111 MPa
According ES	s = 6 mm	0,137 MPa